Roll Number		

SET B

INDIAN SCHOOL MUSCAT
FINAL TERM EXAMINATION
MATHEMATICS

CLASS: XII | Sub. Code: 041 | Time Allotted: 3 Hrs

26.11.2018 | Max. Marks: 100

General Instructions:

(i) All questions are compulsory.
(ii) This question paper contains 29 questions.
(iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
(iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
(v) Question 13-23 in Section C are long-answer type questions carrying 4 marks each.
(vi) Question 24-29 in Section D are long-answer type questions carrying 6 marks each.

SECTION A

1. Find the value of $\sin^{-1}[\sin\left(\frac{2\pi}{3}\right)]$. 1

2. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ *and* $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$. 1

3. Find the angle between the two planes 2x + y -2z=5 and 3x -6y -2z =7.

OR 1

Find the angle between the line $\frac{x-2}{3} = \frac{y+1}{1} = \frac{3-z}{-2}$ and the plane $3x + 4y + z = 5$.

4. A random variables X has the following probability distribution 1

X	0	1	2	3	4
P(X)	0	3k	2k	2k	3k

Find the value of k

SECTION B

5. A balloon which is always remains spherical, has a variable radius. Find the rate at which its volume is increasing w.r.t its radius when the radius is 5cm. 2

OR

Find the approximate value of f(5.001) ,where f(x) = $x^3 - 7x^2 + 15$.

6. Find the values of k ,if the function f(x) = $\begin{cases} kx+1, & x \le \frac{\pi}{2} \\ \cos x, & x > \frac{\pi}{2} \end{cases}$ is continuous $at\ x = \frac{\pi}{2}$ 2

7. Show that f(x) = $|x+5|$ is not differentiable at x= -5 2

8. Evaluate: $cos\left(\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{4}{5}\right)$ 2

9. Form the differential equation representing the family of curves $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $a\ and\ b$ are arbitrary constants. 2

10. There are two types of fertilizers $F_1$and F_2 . F_1consists of 10% of nitrogen and 6% phosphoric acid and F_2 consists of 5% of nitrogen and 10% phosphoric acid .After testing the soil conditions a farmer finds that she needs at least 14 kg of nitrogen and 14kg of phosphoric acid for her crops, If F_1 costs Rs6/kg and F_2 costs Rs5/Kg. Formulate the problem so that nutrient requirements are met at a minimum cost. 2

OR

An aeroplane can carry a maximum of 200 passengers. A profit of ₹ 500 is made on each executive class ticket out of which 20% will go to the welfare fund of the employees. Similarly a profit of ₹ 400 is made on each economy class ticket out of which 25% will go for the improvement of facilities provided to the economy class passengers. In both the cases, the remaining profit goes to the airliner's fund. The airline reserves at least 20 seats for executive class. However at least four times as many passengers prefer to travel by economy class than by the executive class. Formulate the problem in order to maximize the net profit of the airline.

11. Find a unit vector perpendicular to each of the vectors, $\vec{a}=\hat{\imath}-7\hat{\jmath}+7\hat{k}\ and\ \vec{b}=3\hat{\imath}-2\hat{\jmath}+2\hat{k}.$ 2

OR

Given $|\vec{a}|=13, |\vec{b}|=5, and\ \vec{a}.\vec{b}=60\ .find\ |\vec{a}\ x\vec{b}|$.

12. If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? 2

SECTION C

13. Solve : $\cot^{-1}x-\ \cot^{-1}(x+2)=\ \frac{\pi}{12}$, x$>0$ 4

OR

Prove that $\sin^{-1}\left(\frac{5}{13}\right)$ + $\cos^{-1}\left(\frac{3}{5}\right)$ = $\tan^{-1}\frac{63}{16}$

14. Verify Mean value theorem for the function, f(x) = x -2sinx on [-π,π] 4

OR

If $x^{16}y^9 = (x^2+y)^{17}$. Prove that $\frac{dy}{dx}=\frac{2y}{x}$.

15. If x = asin2t (1+cos2t) and y = bcos2t(1-cos2t) , find $\frac{dy}{dx}$ at t = $\frac{\pi}{4}$. 4

16. Find the area of the region bounded by the two parabolas $y = x^2$ *and* $y^2 = x$,using integration. 4

17. Find the points on the curve $9y^2 = x^3$ where the normal to curve makes equal intercepts with the axes. 4

OR

Find the equation of the tangent to the curve y = $\sqrt{5x-3}$ - 2 which is parallel to the line 4x – 2y + 3 = 0.

4

18. Find τ if the vectors $\vec{a} = \hat{\imath} + \hat{3j} + \hat{k}$, $\vec{b} = \hat{2i} - \hat{\jmath} - \hat{k}$ and $\vec{c} = \tau\,\hat{\imath} + 3\hat{k}$ are coplanar.

19. Find the shortest distance between the pairs of lines given by

$$\vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda\left(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}\right) and\ \vec{r} = 2\hat{\imath} + 4\hat{\jmath} + 5\hat{k} + \mu(3\hat{\imath} + 4\hat{\jmath} + 5\hat{k})$$ 4

20. A Factory has three machines X, Y and Z producing 1000, 2000 and 3000 bolts per day respectively. The machine X produce 1% defective bolts, Y produces 15% defective bolts and Z produce 2% defective bolts. At the end of the day, a bolt is drawn at random and is found defective. What is the probability that this defective bolt has been produced by the machine X. 4

21. If $\hat{a}$ and $\hat{b}$ are unit vectors inclined at an angle θ, then prove that 4

$$\sin\frac{\theta}{2} = \frac{1}{2}\left|\hat{a} - \hat{b}\right|.$$

22. Find the intervals in which the functions given below are strictly decreasing or strictly increasing:- 4

$$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$$

23. Find the general solution of the differential equation

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2).$$ 4

SECTION D

24. Find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ *and* $(x-2)^2 + y^2 = 4$. 6

OR

Using integration find the area of region bounded by the triangle whose vertices are (1,0),(2,2) and (3,1).

25. Show that the altitude of a right circular cone of maximum volume that can be inscribed in a sphere of radius R is $\frac{4R}{3}$. 6

26. Solve the differential equation: $(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}\ given\ y = 0\ when\ x = 1.$ 6

OR

Solve: $(x^3+x^2+x+1)\ \frac{dy}{dx} = \ 2x^2 + x; y = 1\ when\ x = 0$

27. Find the equation of the plane through the intersection of the planes x –2y+3z-4=0 and x +2y +3z-1=0 and passing through the point (-1,2,-1) 6

OR

Find the length and the foot of the perpendicular from the point (1,3,4) to the plane 2x – y+z+3=0 , also find image point.

28. Two Cards are drawn Simultaneously (or successively without replacement) from a well shuffled deck of 52 cards. Find the means variance and standard deviation of the number of Aces. 6

29. A manufactures produces nut and bolts. It takes 1 hour of work on machine A and 3 hours on Machine B to produce of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce of bolts. He earns a profit of Rs17.50 per package on nuts and Rs7.000 per package on bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates his machines for at the most 12 hours a day? 6

End of the Question Paper